Problem 14

A computer program gives the result $1/6$ for the sum of the series $\sum_{n=0}^{\infty}(-5)^n$. Show that this series is divergent. Do you see what happened? *Warning hint:* Always consider whether an answer is reasonable, whether it's a computer answer or your work by hand.

Solution

The series is considered to be divergent because the sum doesn't evaluate to any particular value.

$$\sum_{n=0}^{\infty}(-5)^n = (-5)^0 + (-5)^1 + (-5)^2 + (-5)^3 + (-5)^4 + (-5)^5 + \cdots$$

$$= (1) + (-5) + (25) + (-125) + (625) + (-3125) + \cdots$$

$$= ?$$

The computer program foolishly used the infinite summation formula,

$$\sum_{n=0}^{\infty} cr^n = \frac{c}{1-r},$$

which only applies if $|r| < 1$.

$$\sum_{n=0}^{\infty}(-5)^n \neq \frac{1}{1-(-5)} = \frac{1}{6}$$